# Some Aspects of Collision Avoidance

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## Theme

SOME aspects of the problem of collision avoidance for two vehicles are examined by hypothesizing certain rationales for the vehicle operators and then computing the sets of initial conditions for which collision can occur. We first suppose that one operator may be unaware of any danger and so may control his vehicle in a hazardous fashion. To this operator we assign the role of pursuer, while the operator of the other vehicle is given the role of evader and a differential game is formulated. The set of initial conditions for which collision can occur despite any maneuvers by the evader is a dangerous (Red) zone. Clearly, a cautious pilot would not like a second vehicle to be in his Red zone, and a good collision avoidance system (CAS) should be effective in this zone. A second zone is defined by assigning a passive role (constant control) to the evader, while the other vehicle still pursues. The set of initial conditions for which no collision is possible is a particularly safe zone since no active avoidance is required. Any practical CAS should not call for unnecessary maneuvers in the (Green) zone. The points in neither the Red nor the Green zones form a Yellow zone in which some evasive maneuver may be required. A workable CAS should be active for initial points in the Yellow zone, so that the Red zone can be avoided.

## Contents

In order to keep the problem solution and results tractable it is necessary to employ a rather simple model to describe the motions of the vehicles. Accordingly, we assume that each vehicle moves with constant speed in a fixed horizontal plane and has a fixed maximum turning rate (or minimum turning radius). Consider the pursuer, $P$, and evader, $E$ located in the horizontal plane as shown in Fig. 1. The pursuer is moving with constant speed $S_p$ while turning at a rate $u$. The evader is moving with a constant speed $S_E$ while turning at a rate $v$. The instantaneous velocity vectors are as indicated with $u$, $v$, and $x_3$ shown positive.

We will assume that the evader is the faster aircraft and set $S_E = 1$, $S_p = \alpha < 1$. It is further assumed that the maximum turning rate of the evader is less than the pursuer with $v_{max} = 1$, $u_{max} = \delta > 1$. The kinematical equations, in nondimensional form, become

$$\dot{x}_1 = \alpha \sin x_3 + v x_2 \tag{1}$$

$$\dot{x}_2 = \alpha \cos x_3 - 1 - v x_1 \tag{2}$$

$$\dot{x}_3 = v - u \tag{3}$$

$$|u| \leqq \delta, \quad |v| \leqq 1 \tag{4}$$

where the dot denotes differentiation with respect to nondimensional time $t$. The coordinates $x_1$ and $x_2$ must be multiplied by $S_E/v_m$ to obtain length dimensions and $t$ must be multiplied by $1/v_m$ to obtain time. Note that $u = +\delta$, $u = 0$, $u = -\delta$ is a hard left turn, straight line flight, and a hard right turn for the pursuer. Similarly $v = +1$, $v = 0$, $v = -1$ is a hard left turn, straight line flight and a hard right turn for the evader. It will be assumed that collision occurs if the pursuer moves to within a nondimensional radial distance $R$ of the evader. Collision will include the case of tangential encounter at a radius $R$. In the three-dimensional state space the collision surface is defined to be a cylinder of radius $R$ of the evader whose axis is $x_3$.

*The Red zone—a qualitative differential game:* Qualitative games were first discussed by Isaacs[1] and have been investigated extensively by Blaquiere, et al.[2] In the nomenclature of Ref. 2, the solution of the game involves determining certain game surfaces in the state space. The characteristic feature of the game surface is the following: if the optimal control pair ($u^*$, $v^*$) give rise to a trajectory on the game surface, then nonoptimal play by one player cannot cause the system to move into a zone desirable for him. For example, if $P$ uses nonoptimal control $u$ on the game surface and $E$ uses an optimal control $v^*$, then the system cannot move into the Red zone, and in general it will move into the noncapturable zone. For our problem there are two game surfaces emanating from the collision surface which intersect and the enclosed points are the Red zone.

Necessary conditions for construction of the game surfaces are derived in Ref. 2. The approach used was to synthesize controls from the necessary conditions and then integra.e the system equations backwards from the collision surface. The necessary conditions also characterize the points on the terminal surface from which game surface trajectories emanate (in the retrograde sense). The details of the analysis may be found in Ref. 3. Here we only point out that game surface controls for the evader are bang-bang, while the pursuer may also use "singular" control, in this case $u = 0$ (i.e., straight line motion). Additionally

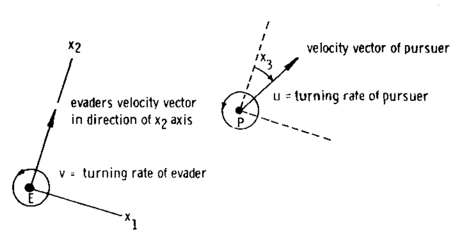


**Fig. 1 Pursuer and evader moving in the horizontal plane.**

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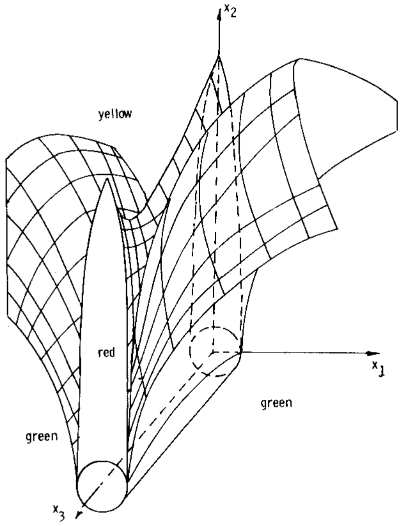


Fig. 2 Qualitative sketch of the Red and Green barriers.

Meier,[4] has considered a more involved analysis to determine time optimal strategies from points within the Red zone.

*The Green zone—a problem in controllability:* If we now put $v = 0$ in Eqs. (1–3), we have the system with the "Evader" executing straight line motion. The set of points for which $P$ cannot cause a collision is the Green zone, and its determination hinges on the construction of a "Controllable Surface." Such a surface locally divides the space into capturable and noncapturable regions. Necessary conditions for construction of the controllable surface are given in Ref. 5 (also see Ref. 6). Again the method of construction was retrograde integration of the system equations. Details for the stated problem are given in Ref. 3.

The theoretical results outlined above may now be used to gain some insight into the problem of collision avoidance. For the simple model used we have been able to divide the state space into three zones: 1) Green Zone—If the evader continues in his current direction collision is not possible. 2) Yellow Zone—The evader can avoid collision but if he persists in his current heading collision is possible. 3) Red Zone—Despite any maneuvers by the evader collision is possible.

Figure 2 gives a general representation of these zones and Fig. 3 presents a constant $x_3$ cross section of the zones for the case $\alpha = 0.5$, $\delta = 2.5$ and $R = 0.02$. These are values typical of a

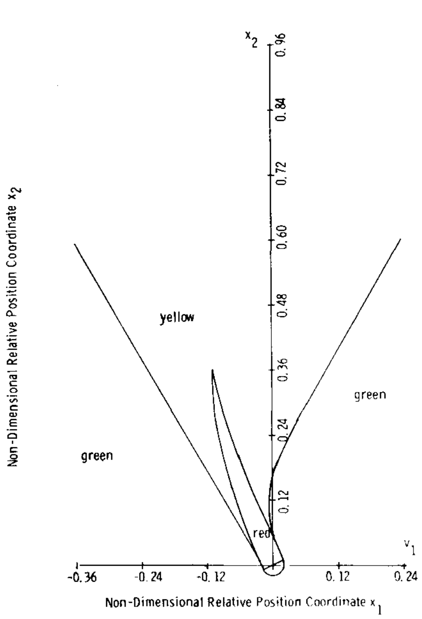


Fig. 3 Red and Green barrier cross section at $x_3 = 30°$.

light aircraft (Pursuer) and a commercial aircraft (Evader) in the vicinity of an airport.

Although the preceding analysis affords valuable information about zones of vulnerability it apparently provides little information about evasive maneuvers. Indeed, for qualitative games, optimality only has meaning for trajectories on the game surfaces. It is possible, however, to deduce reasonable evasive strategies by considering a family of such games with differing capture radii. Figure 4 is a typical cross section resulting from a family of such solutions. Any point in the state space can lie on some such Red barrier. The $R$ value corresponding to the barrier through a particular point is the minimax of the closest approach distance. To utilize these results the evader would track neighboring aircraft and if one got inside the barrier for some threshold $R$ value, evasive action would be taken. Evasive strategies from points within the Yellow zone are characterized in Ref. 3. A worst case collision zone for an unknown pursuer heading ($x_3$) may be found by taking the union of the Red zones $x_3$-cross sections, as $x_3$ varies from 0 to $2\pi$. Similarly, the common intersection of all Green zones $x_3$-cross sections ($0 \leq x_3 < 2\pi$) is the region in which straight line evader motion is safe, regardless of the relative heading.

Finally, the utility of modeling collision avoidance as a pursuit-evasion game requires some comment. The rationale, of course, is to perform a worst case analysis and determine a zone wherein nonoccurrence of a collision can be guaranteed by the evader. It might be more reasonable to restrict the pursuer to open loop controls so that he would fly a hazardous path but not behave like an "optimal kamikaze." In this regard it should be emphasized that the results outlined above have been synthesized largely from necessary conditions.

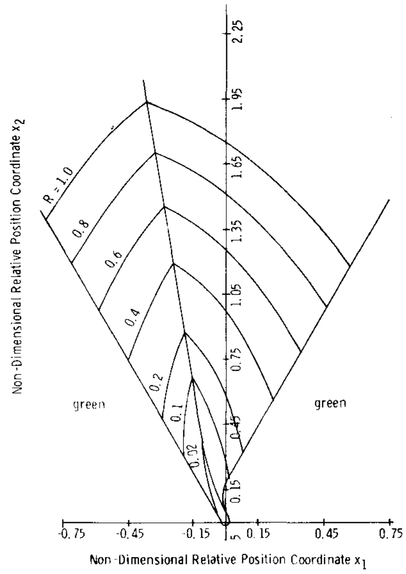


Fig. 4 Strategy for the Yellow zone at $x_3 = 30°$.

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